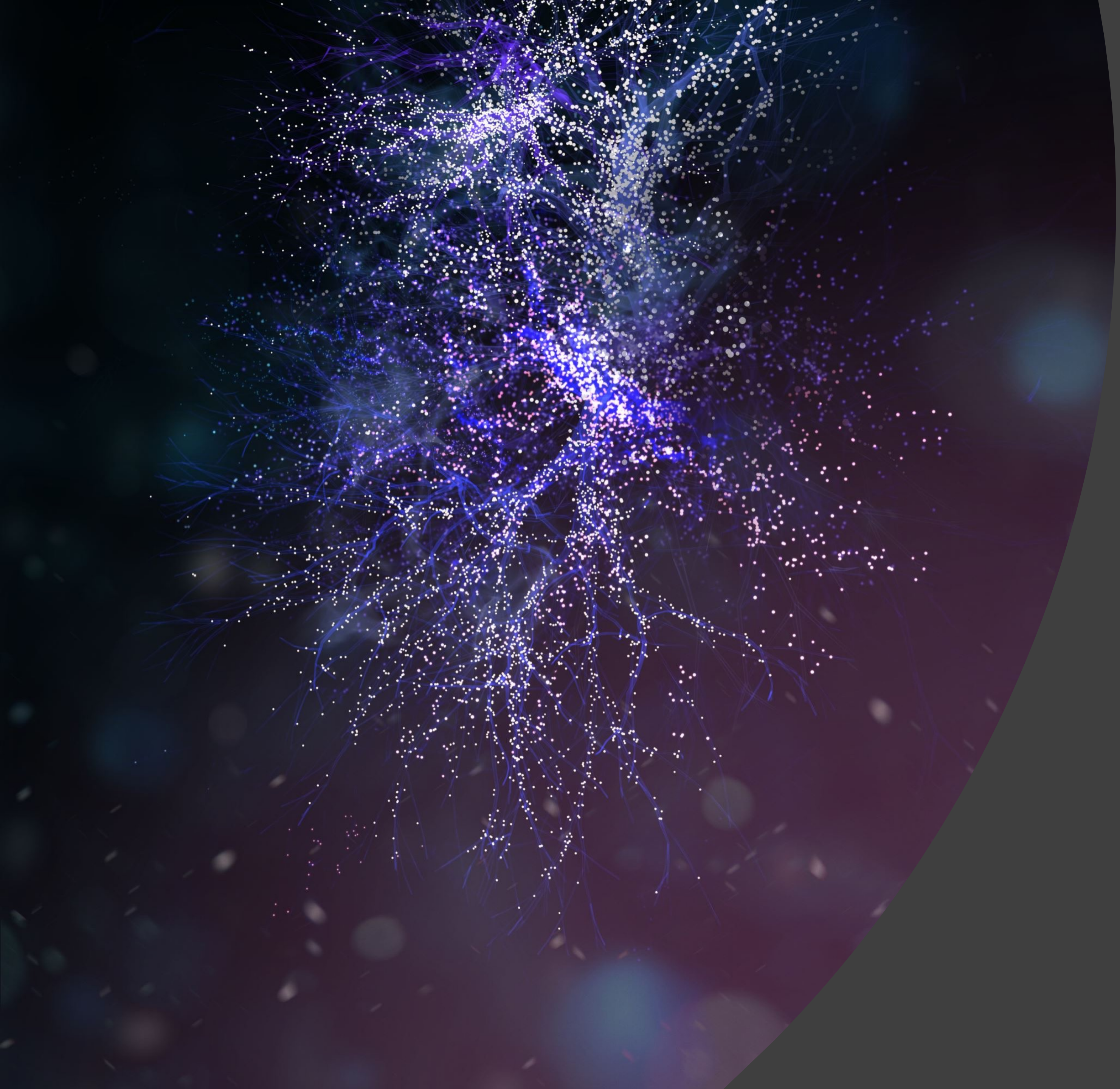


Can the Shape of Our Universe Explain the Dark Matter?

Student: Hang Su

Mentors:

Dr. Nikodem Poplawski

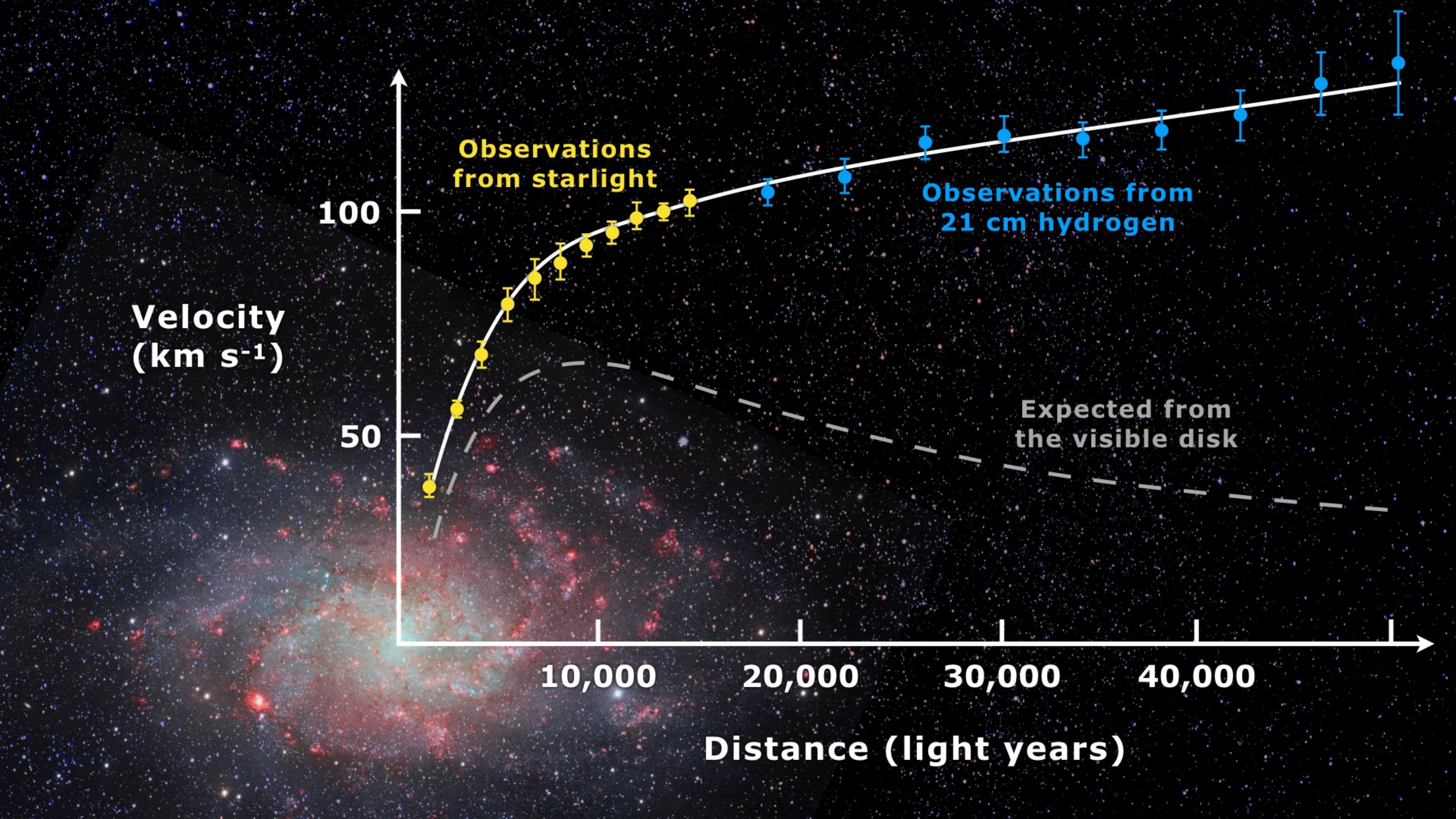
Dr. Kevin Green

Overview

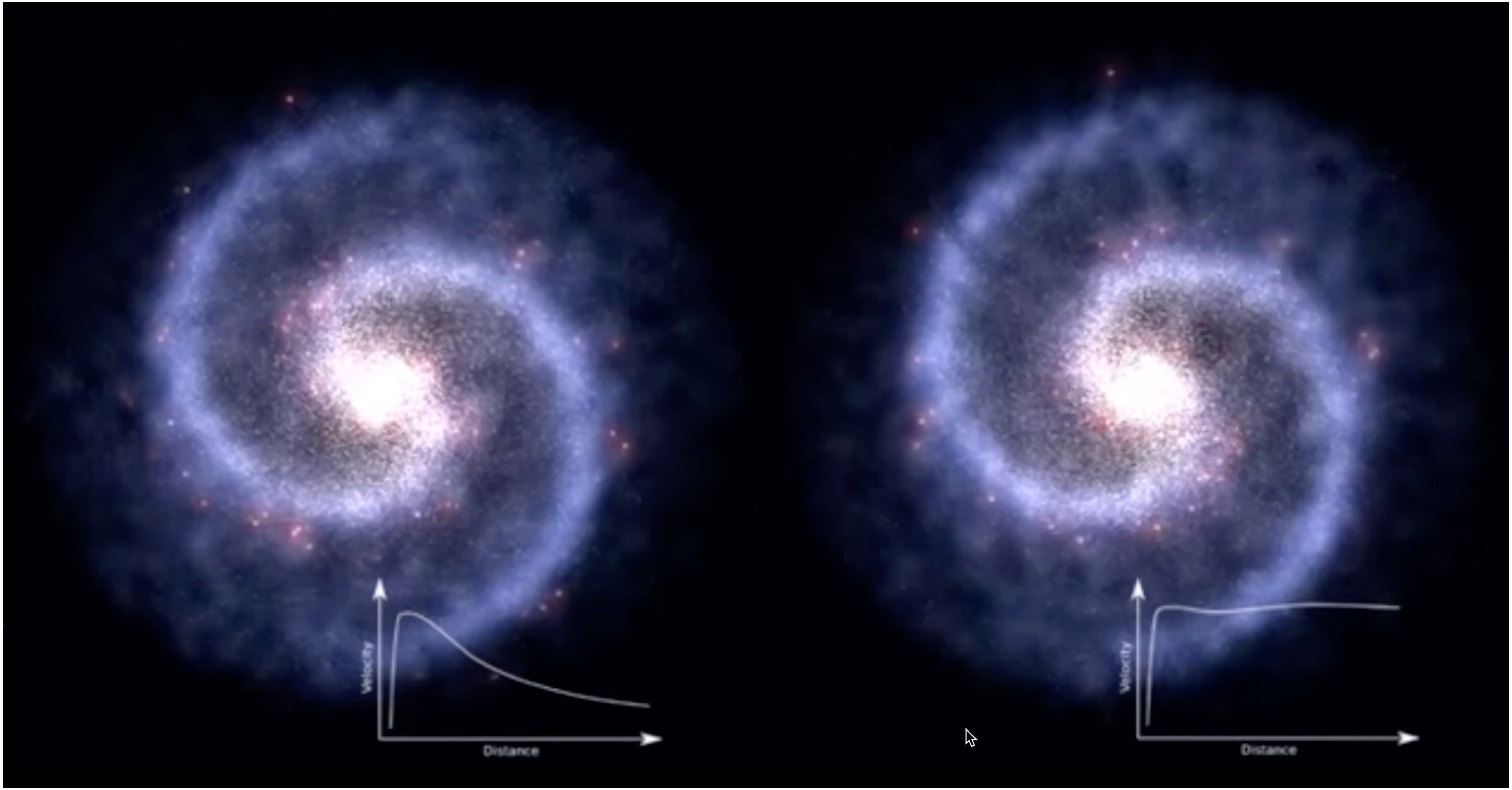
- Dark matter
- Dark energy
- Shape of the universe
- Stereographic projection

Dark Matter

- 85% of the matter in the universe.
- it does not appear to interact with the electromagnetic field
- How it was discovered: Galaxy Rotation Curves

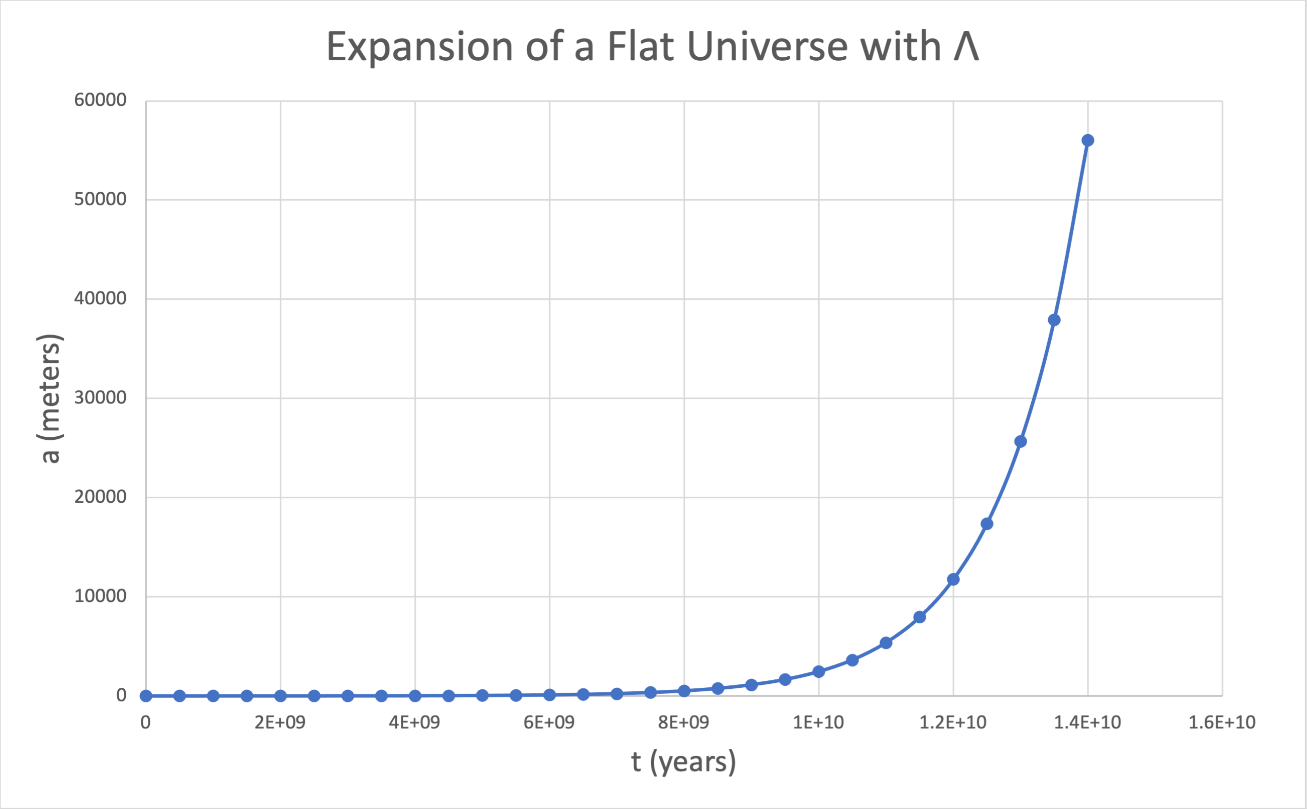


Observations
from starlight
Observations from
21 cm hydrogen
Velocity
(km s-1)
100
50
Expected from
the visible disk
10,000
20,000
30,000
40,000
Distance (light years)


Velocity
Distance
Velocity
Distance

Dark Energy (Cosmological Constant Λ)

- Dark energy is the name given to the force that is believed to accelerate the expansion of the universe. Distant galaxies appear to be moving away from us at high speed: the idea is that the universe is getting bigger and has been since the Big Bang.



$$\frac{da}{dt} = c\sqrt{\frac{\Lambda}{3}}a, \quad a = a(0)\exp\left(\sqrt{\frac{\Lambda}{3}}ct\right) = a(0)e^{Ht}$$

Empty universe with dark energy expands exponentially.

We hypothesize dark energy comes from the shape of the universe.

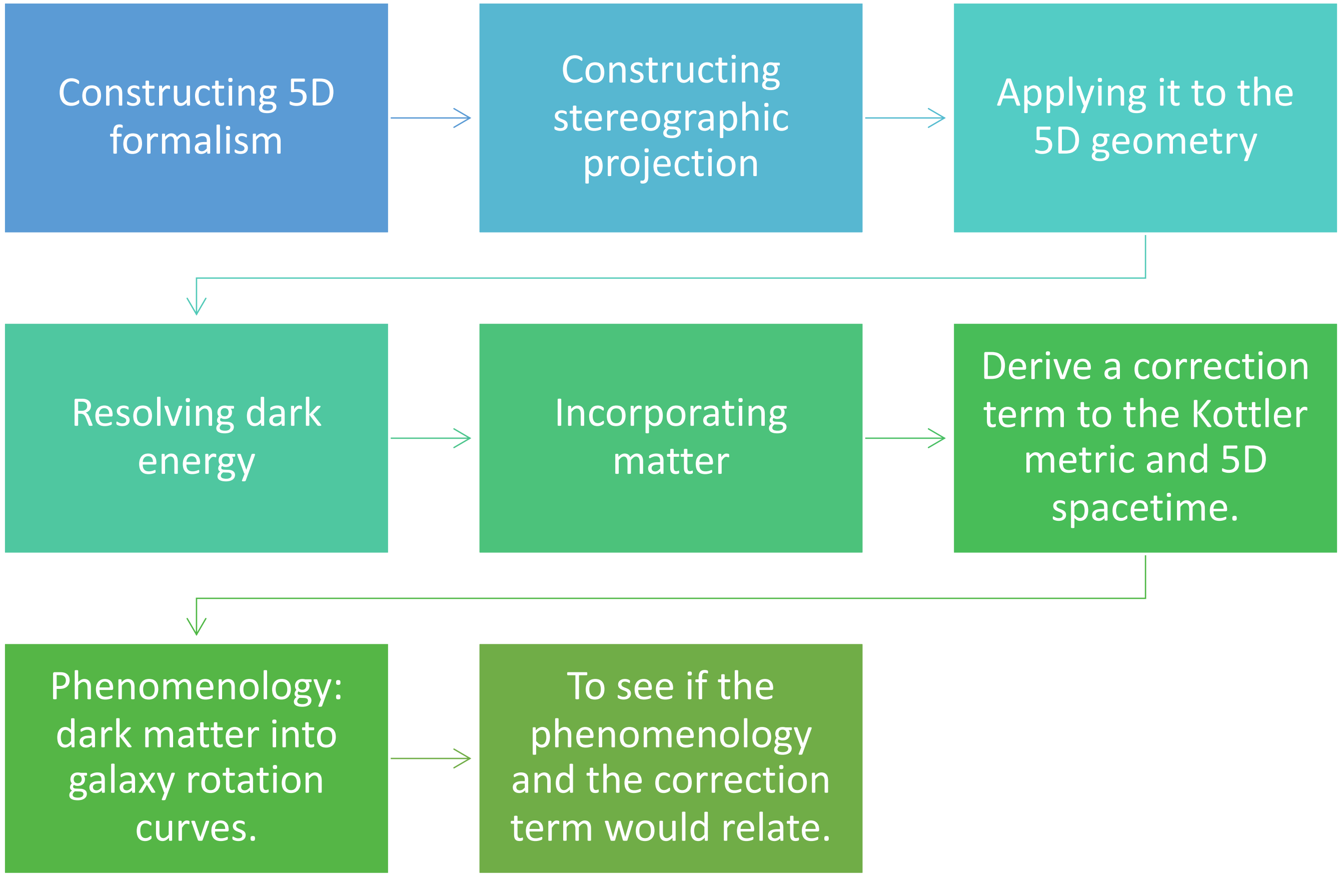
Stereographic Projection

- Space points on the two-dimensional surface of a three-dimensional sphere, which is a surface with constant curvature, can be projected on a plane tangent to the sphere. This plane is referred to as a stereographic plane.

Purpose

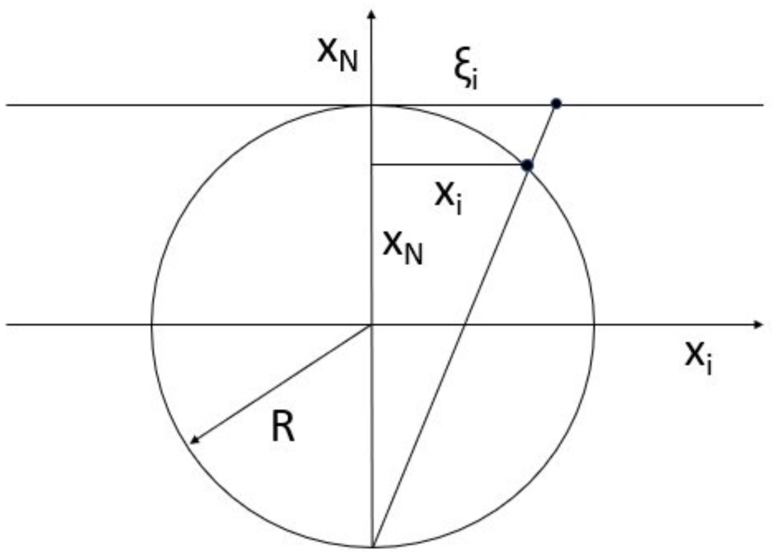
- To explain the nature of dark matter and dark energy using higher dimensional analysis.

Methods


Constructing 5D formalism
Constructing stereographic projection
Applying it to the 5D geometry
Resolving dark energy
Incorporating matter
Derive a correction term to the Kottler metric and 5D spacetime.
Phenomenology: dark matter into galaxy rotation curves.
To see if the phenomenology and the correction term would relate.

Preliminary Results

$$\sum_i x_i x_i + x_N x_N = R^2$$



Stereographic Coordinates

- A point with coordinates xi, xN (i = 1, 2, N = 3) lies on a sphere with radius R.

The coordinates on the sphere and the stereographic coordinates on the plane are related by:

$$x_i = \frac{\xi_i}{1 + \xi^2/4R^2}, \quad x_N = R\,\frac{1 - \xi^2/4R^2}{1 + \xi^2/4R^2}$$

$$\xi^2 = \sum_i \xi_i \xi_i$$

Empty universe looks like surface of 5D sphere with radius R in 5D pseudo-Euclidean, flat space:

$$\eta_1^2 + \eta_2^2 + \eta_3^2 - \eta_4^2 + \eta_5^2 = R^2$$

Stereographic and 5-Dimensional

$$\eta_i = \frac{\xi_i}{1+\xi^2/4R^2}, \qquad \eta_5 = R\,\frac{1-\xi^2/4R^2}{1+\xi^2/4R^2}$$

De Sitter Metric of Space time

- De Sitter universe: empty expanding universe
- c: speed of light
- t: time
- a: scale factor
- r: radius of the universe
- θ, Φ : spherical coordinates

$$ds^2 = c^2 dt^2 - a^2(0)e^{2Ht}(dr^2 + r^2 d\theta^2 + r^2 \sin^2\theta \, d\phi^2)$$

Dark Energy

- Applying a coordinate transformation of de Sitter metric:

$$ds^2 = c^2dt^2 - a^2(0)e^{2Ht}(dr^2 + r^2d\theta^2 + r^2\sin^2\theta\, d\phi^2)$$

↓

$$ds^2 = \left(1 - \frac{1}{3}\Lambda r^2\right)c^2dt^2 - \left(1 - \frac{1}{3}\Lambda r^2\right)^{-1}dr^2 - r^2d\theta^2 - r^2\sin^2\theta\, d\phi^2$$

↓

$$d\eta_1^2 + d\eta_2^2 + d\eta_3^2 - d\eta_4^2 + d\eta_5^2 = -\left(1 - \frac{r^2}{R^2}\right)c^2dt^2 + \left(1 - \frac{r^2}{R^2}\right)^{-1}dr^2 + r^2d\theta^2 + r^2\sin^2\theta\, d\phi^2 = -ds^2$$

$$R = \left(\frac{3}{\Lambda}\right)^{1/2}$$

Dark energy is eliminated from the geometry.

Kottler Metric of Space time

- Now we add matter!
- Kottler universe: expanding universe with one blackhole.
- c: speed of light
- t: time
- r: radius of the universe
- θ, Φ : spherical coordinates
- Λ: cosmological constant
- rg: Schwarzschild radius

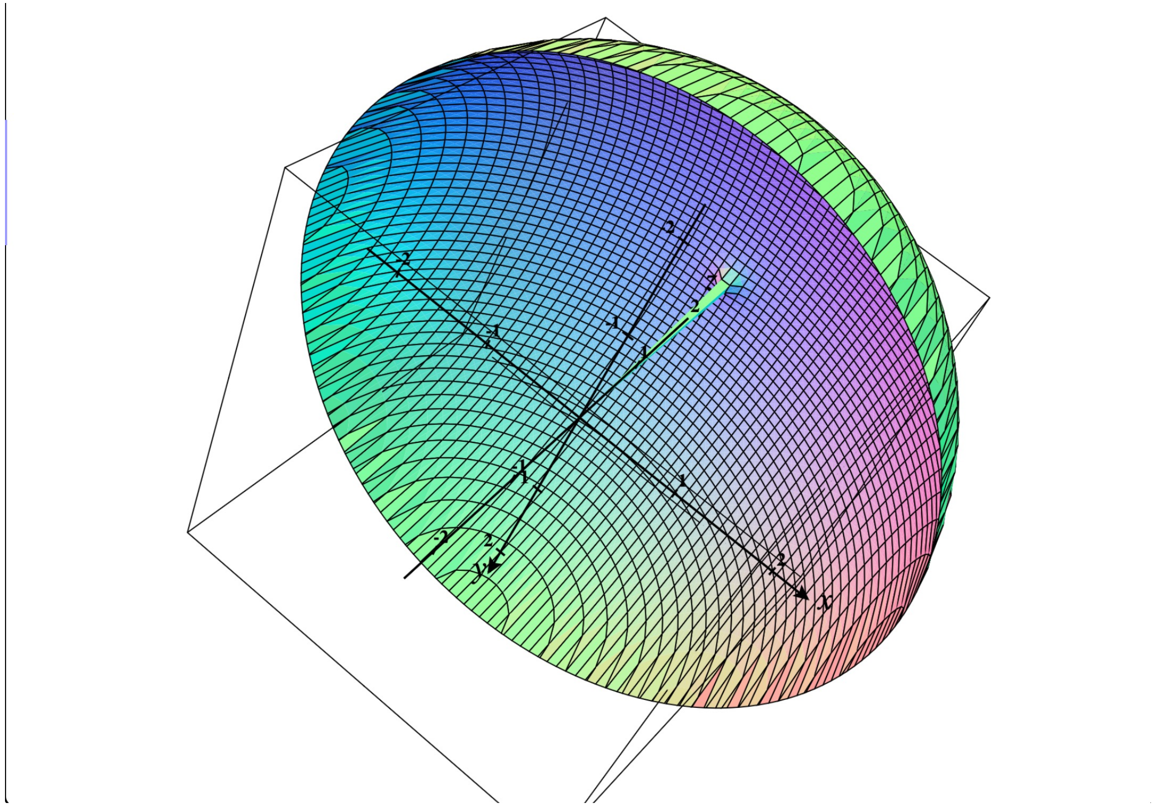
$$ds^2 = \left(1 - \frac{r_g}{r} - \frac{1}{3}\Lambda r^2\right)c^2 dt^2 - \left(1 - \frac{r_g}{r} - \frac{1}{3}\Lambda r^2\right)^{-1} dr^2 - r^2 d\theta^2 - r^2 \sin^2\theta \, d\phi^2$$

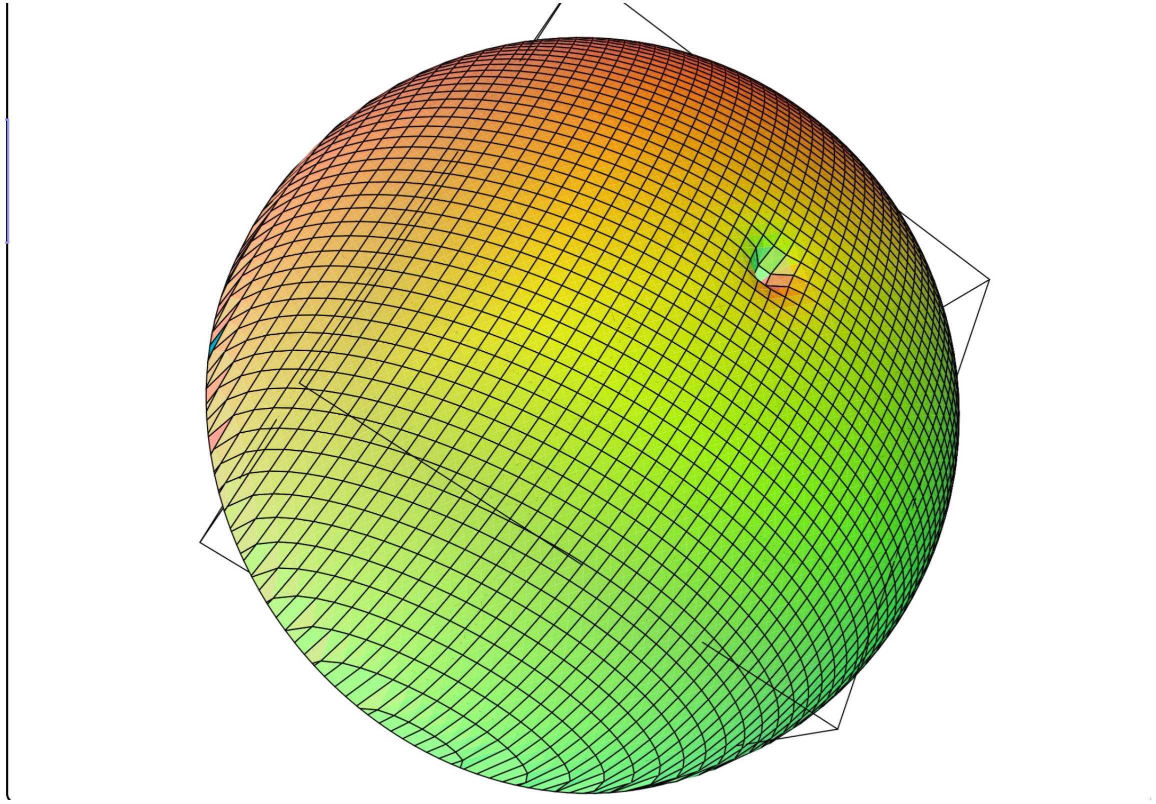
Prove: Kottler universe is a 4D surface of a 5D deformed sphere in not flat 5D space.

$$\eta_1^2 + \eta_2^2 + \eta_3^2 - \eta_4^2 + \eta_5^2 =$$

$$R^2(1 - \frac{r_g}{r})$$

Effect of one blackhole on the 5D plane

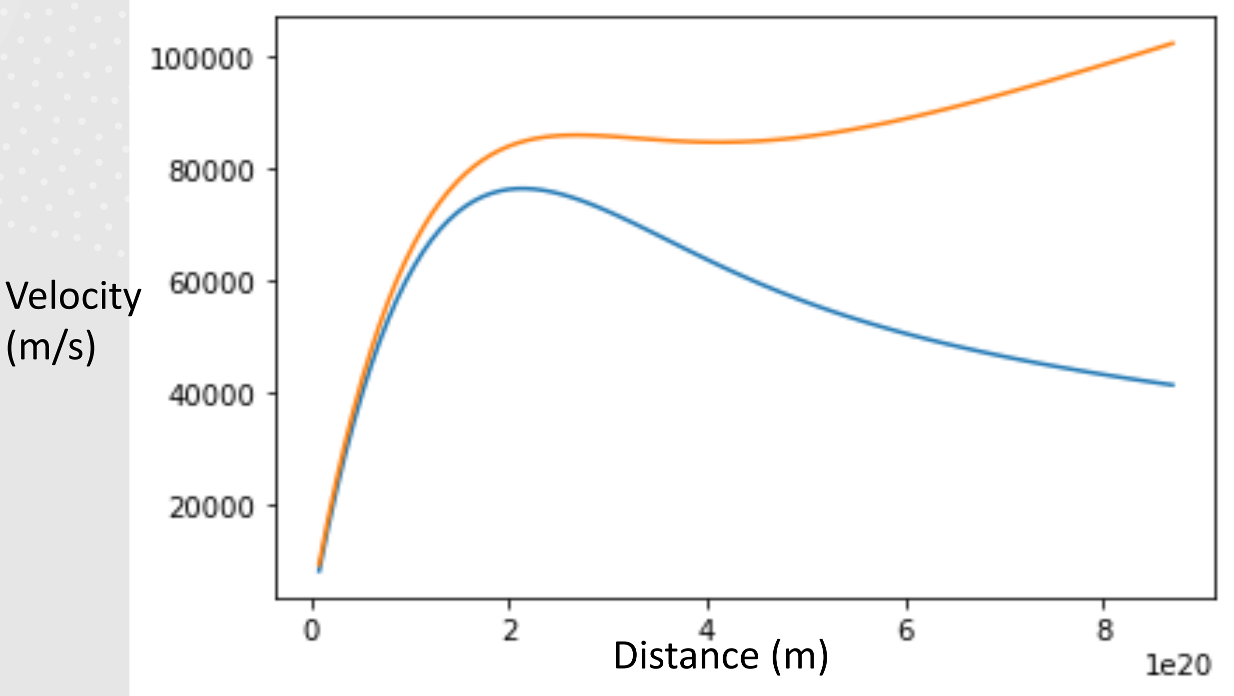


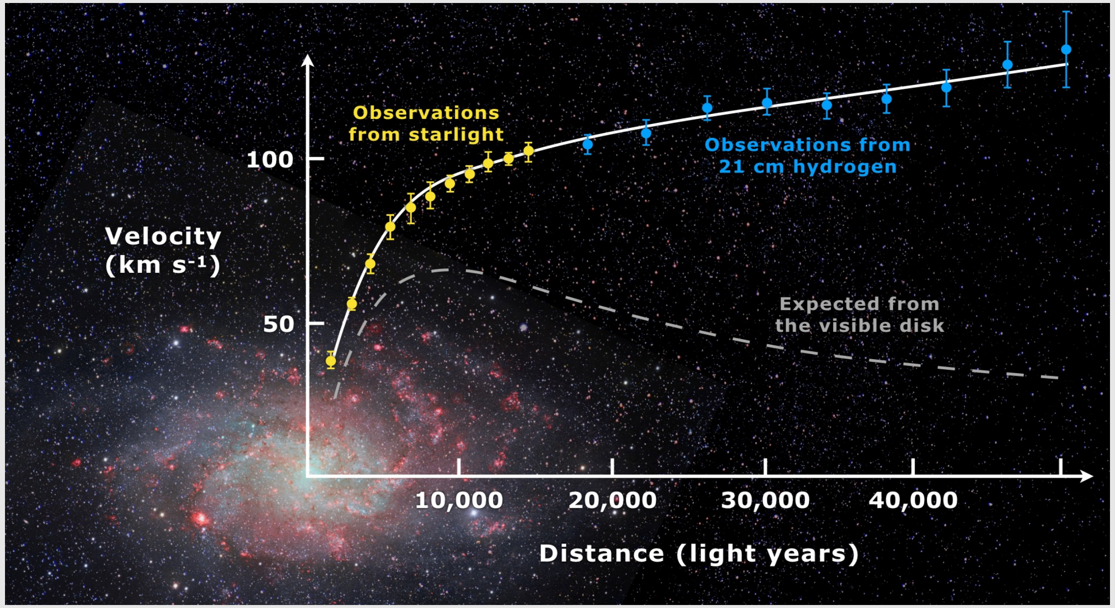


Phenomenology

- Galaxy rotation curves:

$$V^2(r) = r\frac{\partial\Phi}{\partial r} = 4\pi G\int_0^r \frac{k'\rho(a)a^2da}{(r^2 - k^2a^2)^{1/2}}$$





Next Steps

- Continue to find out what would happen if we replace the empty universe with universe with one galaxy. (in progress)
- Derive a correction term to the Kottler metric and 5D spacetime.
- Phenomenology: dark matter into galaxy rotation curves. (in progress)
- To see if the phenomenology and the correction term would relate.
- Refine LaTeX document for SURF article
- To write a paper to submit to a physics or astrophysics journal.

Images and Animations

https://en.wikipedia.org/wiki/Dark_matter

https://en.wikipedia.org/wiki/Galaxy_rotation_curve#cite_note-Rubin1980-15

https://simple.wikipedia.org/wiki/Dark_energy

https://c3d.libretexts.org/CalcPlot3D/index.html

References

Di Valentino, E., Melchiorri, A., Silk, J., “Planck evidence for a closed universe and a possible crisis for cosmology.” Nature Astronomy 4, 196 (2020).

Ferreira, P. G., “Cosmological tests of gravity.” Annual Review of Astronomy and Astrophysics 57, 335 (2019).

Liu, J., Chen, X., Ji, X., “Current status of direct dark matter detection experiments,” Nature Physics 13, 212 (2017).

Lord, E. A., “Tensors, Relativity and Cosmology” (McGraw-Hill 1976).

Milgrom, M. “A modification of the Newtonian dynamics as a possible alternative to the hidden mass hypothesis.” Astrophysical Journal 270, 365 (1983).

Rich, J., “Fundamentals of Cosmology” (Springer, 2001).

Brandt, J. C. (n.d.). *On the Distribution Of mass in Galaxies. I. The Large-Scale Structure of Ordinary Spirals with Applications to M 31.* 1960ApJ...131..293B page 294. http://articles.adsabs.harvard.edu//full/1960ApJ...131..293B/0000294.000.html.

Ryden, B., Peterson, B. M., “Foundations of Astrophysics” (Cambridge University Press, 2020).

Wesson, P. S., “Five-Dimensional Physics: Classical and Quantum Consequences of Kaluza-Klein Cosmology” (World Scientific, 2006).

Nordsieck, K. H. (1973). The angular momentum of spiral galaxies. methods of rotation-curve analysis. The Astrophysical Journal, 184, 719. https://doi.org/10.1086/152364

Bosma, A. (n.d.). The distribution and kinematics of neutral hydrogen in spiral galaxies of various MORPHOLOGICAL types - A. Bosma. http://ned.ipac.caltech.edu/level5/March05/Bosma/frames.html.

Gorbatenko, M. V., Sedov, S. Y. The Mannheim-Kazanas solution, the conformal geometrodynamics and the dark matter. arXiv.org. https://arxiv.org/abs/1711.06189. (2017)

Q&A