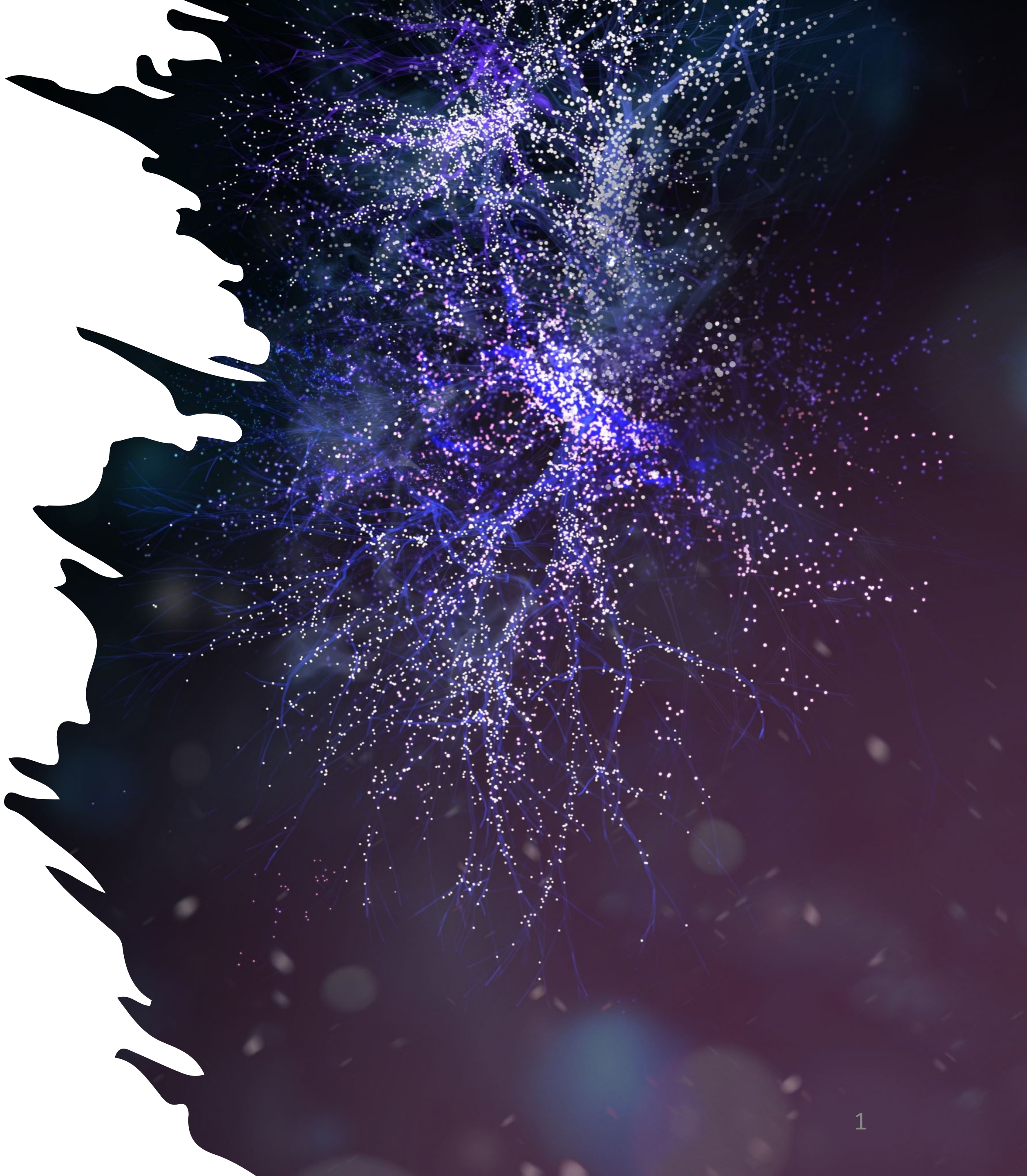
Dark Energy and Dark Matter as Five-Dimensional Stereographic Projection

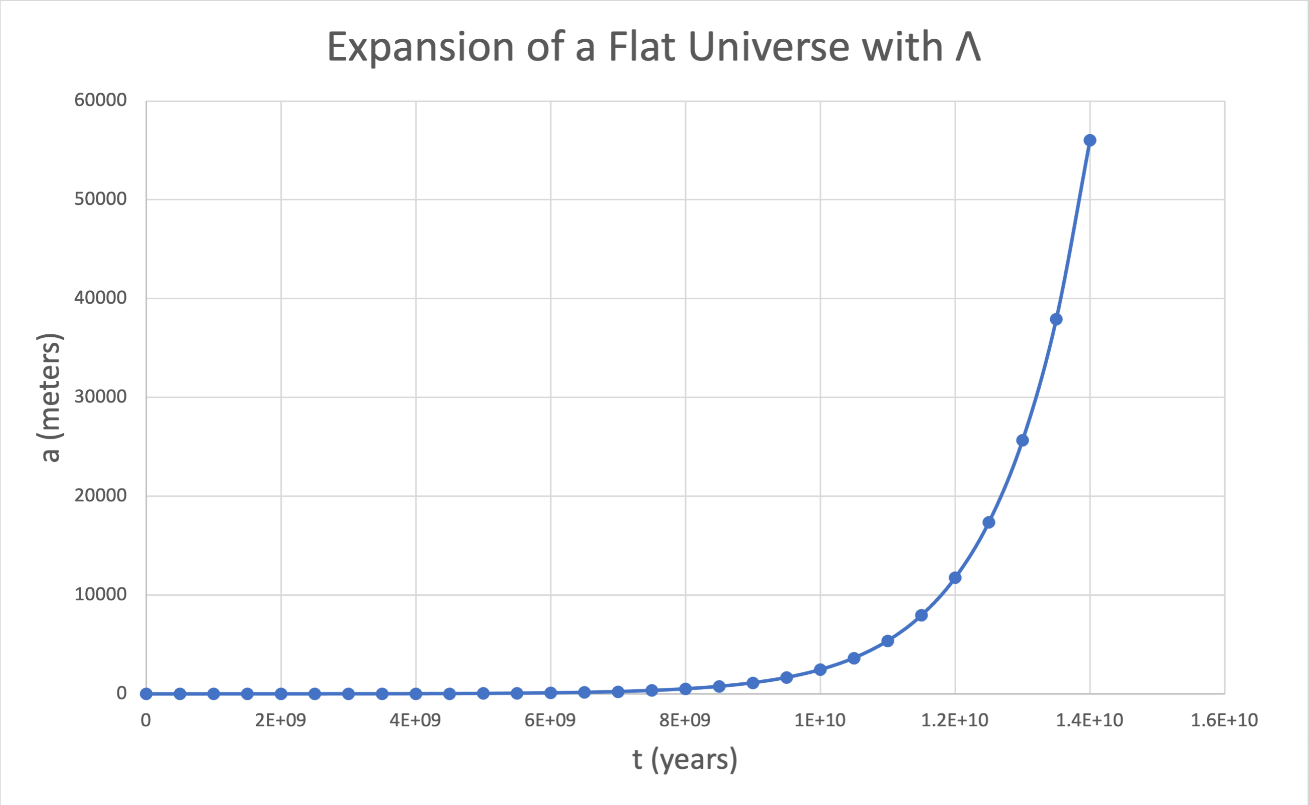
Hang Su

Nikodem Poplawski Ph.D.

Kevin Green Ph.D.

University of New Haven





$$\frac{da}{dt} = c\sqrt{\frac{\Lambda}{3}}a, \quad a = a(0)\exp\left(\sqrt{\frac{\Lambda}{3}}ct\right) = a(0)e^{Ht}$$

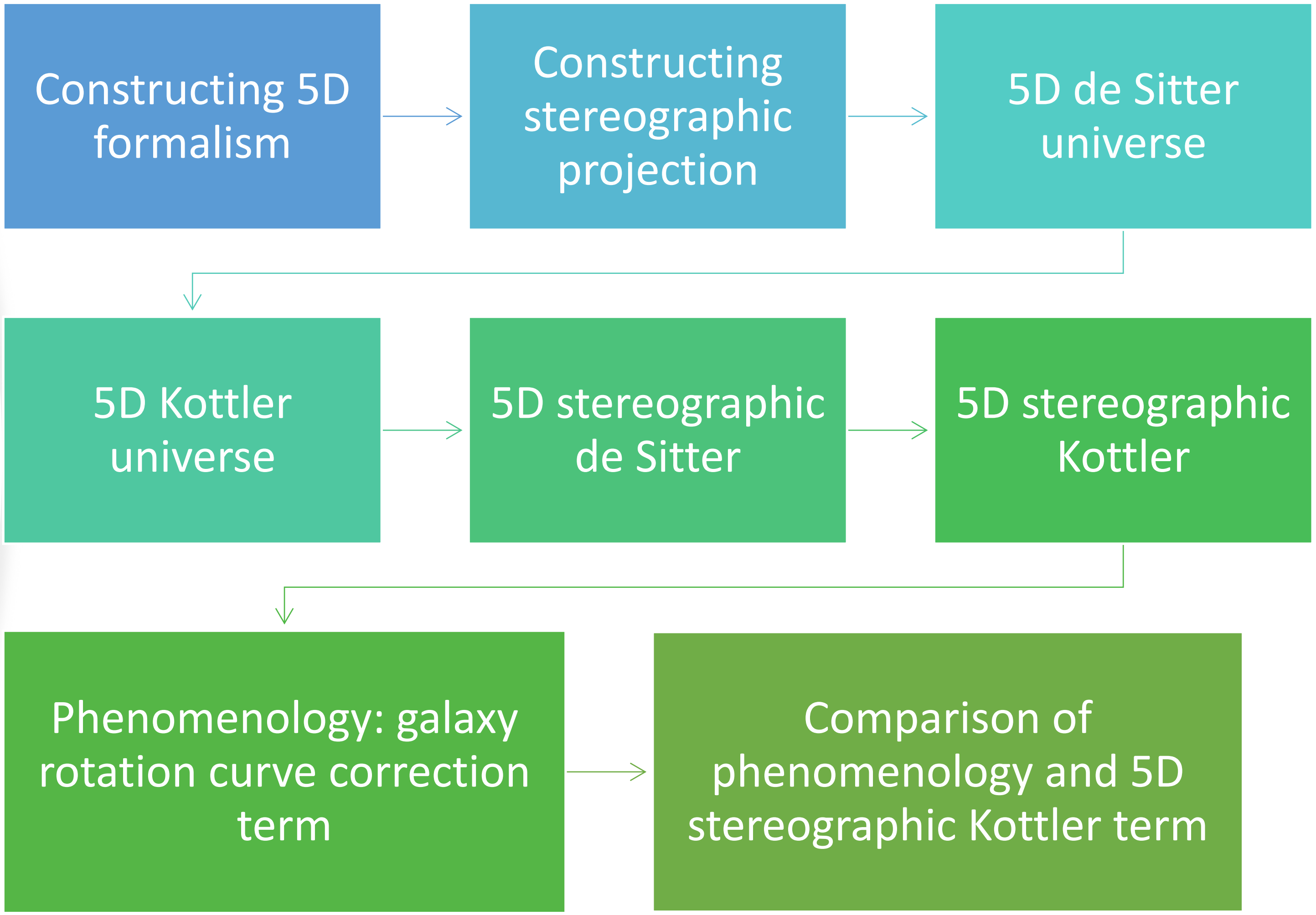
Empty universe with dark energy expands exponentially.

We hypothesize dark energy comes from the shape of the universe.

Hypothesis

Dark matter and dark energy are of the same nature, and they are the product of the universe being a 4D hypersurface on a 5D hypersphere projected onto a 4D hyperplane.

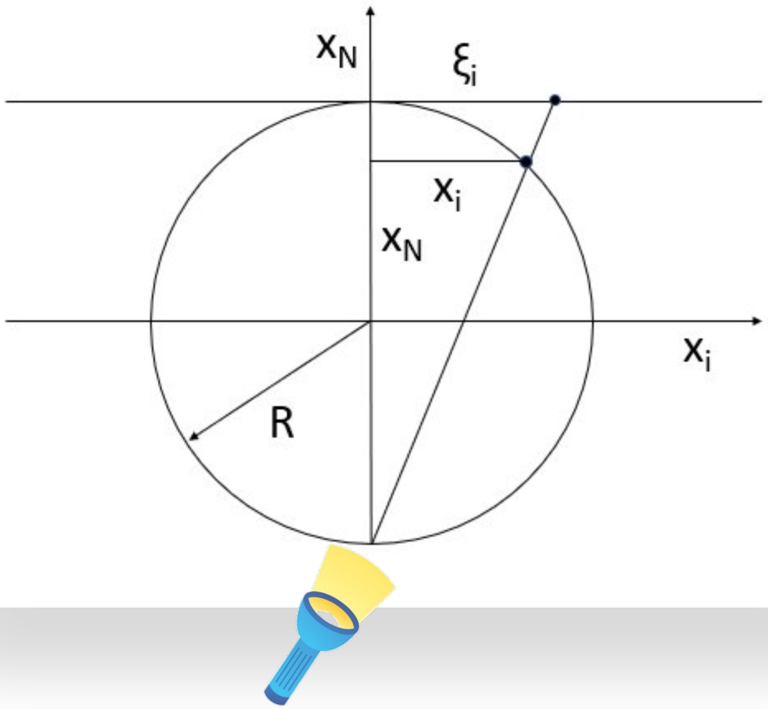
Methods


Constructing 5D formalism
Constructing stereographic projection
5D de Sitter universe
5D Kottler universe
5D stereographic de Sitter
5D stereographic Kottler
Phenomenology: galaxy rotation curve correction term
Comparison of phenomenology and 5D stereographic Kottler term

Preliminary Results

Stereographic Projection

$$\sum_i x_i x_i + x_N x_N = R^2$$



Stereographic Coordinates

- A point with coordinates $x_{i,}\, x_N$ ($i = 1, 2, N = 3$) lies on a sphere with radius R.

$$x_i = \frac{\xi_i}{1 + \xi^2/4R^2}, \qquad x_N = R\,\frac{1 - \xi^2/4R^2}{1 + \xi^2/4R^2} \qquad \xi^2 = \sum_i \xi_i \xi_i$$

Stereographic vs. Isotropic Spherical Coordinates (FLRW metric)

$$dl^2 = \sum_i dx_i dx_i + dx_N dx_N = \boxed{\frac{\sum_i d\xi_i d\xi_i}{(1+\xi^2/4R^2)^2}}$$

$$ds^2 = c^2 dt^2 - \boxed{\frac{a^2(t)}{(1+kr^2/4)^2}}(dr^2 + r^2 d\theta^2 + r^2 \sin^2\theta \, d\phi^2)$$

The force that is believed to accelerate the expansion of the universe.

Dark Energy (Cosmological Constant Λ)

Empty universe looks like surface of 5D sphere with radius R in 5D pseudo-Euclidean, flat space:

$$\eta_1^2 + \eta_2^2 + \eta_3^2 - \eta_4^2 + \eta_5^2 = R^2$$

$$\eta_1 = r\sin\theta\cos\phi, \quad \eta_2 = r\sin\theta\sin\phi, \quad \eta_3 = r\cos\theta,$$

$$\eta_5 \pm \eta_4 = Re^{\pm ct/R}\left(1 - \frac{r^2}{R^2}\right)^{1/2},$$

This relationship gives:

$$\eta_1^2 + \eta_2^2 + \eta_3^2 = r^2, \quad \eta_5^2 - \eta_4^2 = R^2 - r^2$$

de Sitter metric:

$$ds^2 = c^2 dt^2 - a^2(0) e^{2Ht} (dr^2 + r^2 d\theta^2 + r^2 \sin^2\theta \, d\phi^2)$$

Coordinate transformation:

$$e^{2Ht} \to \left(1 - \frac{1}{3}\Lambda r^2\right) e^{2Ht} \qquad a(0) r \to r e^{-Ht}$$

We get:

$$ds^2 = \left(1 - \frac{1}{3}\Lambda r^2\right)c^2 dt^2 - \left(1 - \frac{1}{3}\Lambda r^2\right)^{-1} dr^2 - r^2 d\theta^2 - r^2 \sin^2\theta\, d\phi^2$$

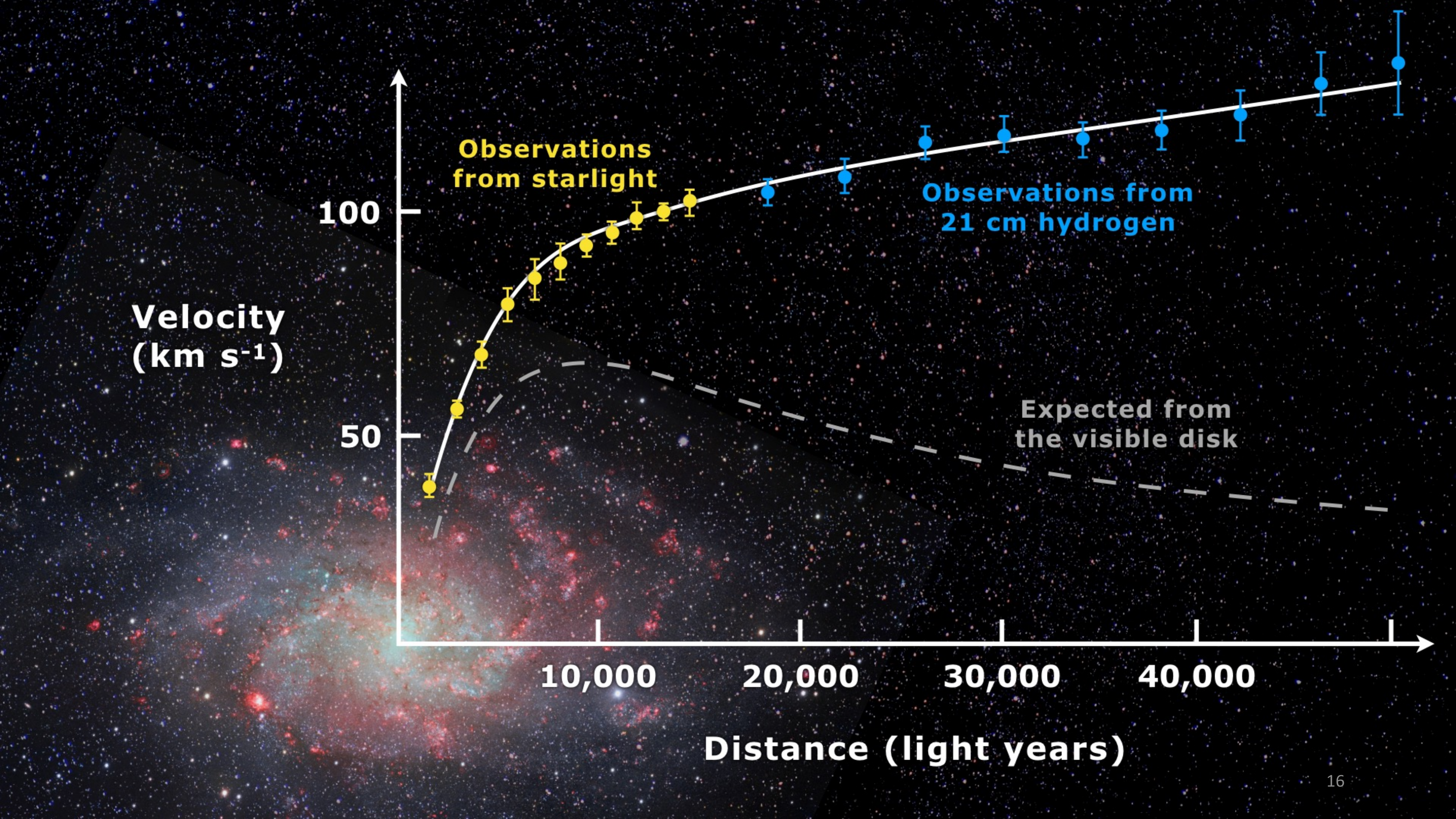
Compared to:

$$d\eta_1^2 + d\eta_2^2 + d\eta_3^2 - d\eta_4^2 + d\eta_5^2 = -\left(1 - \frac{r^2}{R^2}\right)c^2 dt^2 + \left(1 - \frac{r^2}{R^2}\right)^{-1} dr^2 + r^2 d\theta^2 + r^2 \sin^2\theta\, d\phi^2 = -ds^2$$

$$R = \left(\frac{3}{\Lambda}\right)^{1/2}$$

Dark energy can be a nature of the 5D sphere.

Dark Matter


Observations
from starlight
Observations from
21 cm hydrogen
Velocity
(km s-1)
100
50
Expected from
the visible disk
10,000
20,000
30,000
40,000
Distance (light years)

$$ds^2 = \left(1 - \frac{r_g}{r} - \frac{1}{3}\Lambda r^2\right)c^2 dt^2 - \left(1 - \frac{r_g}{r} - \frac{1}{3}\Lambda r^2\right)^{-1} dr^2 - r^2 d\theta^2 - r^2 \sin^2\theta \, d\phi^2$$

Kottler Metric of Spacetime

- Kottler: Schwarzschild-de Sitter universe

Kottler in 5D

$$d\eta_1^2 + d\eta_2^2 + d\eta_3^2 - d\eta_4^2 + d\eta_5^2 = -ds^2 + dr^2\left(\frac{\frac{R^2 r_g^2}{4r^4} - \frac{2r_g}{r}}{1 - \frac{r_g}{r} - \frac{r^2}{R^2}}\right)$$

de Sitter in 5D

$$d\eta_1^2 + d\eta_2^2 + d\eta_3^2 - d\eta_4^2 + d\eta_5^2 = -ds^2$$

Kottler universe is a 4D surface of a 5D deformed sphere in not flat 5D space.

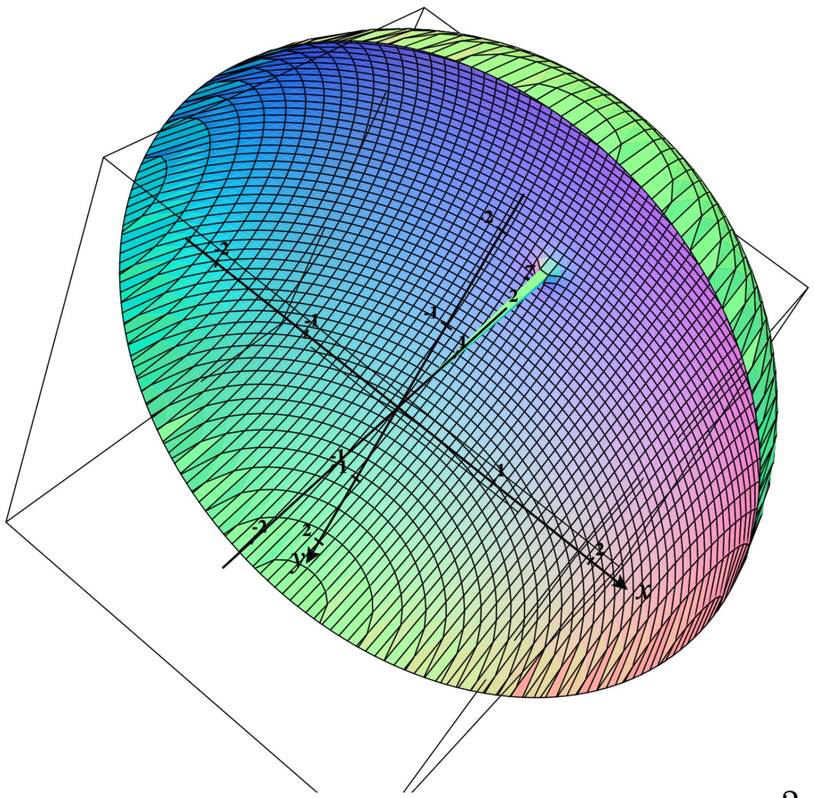
$$\eta_1^2 + \eta_2^2 + \eta_3^2 - \eta_4^2 + \eta_5^2 =$$

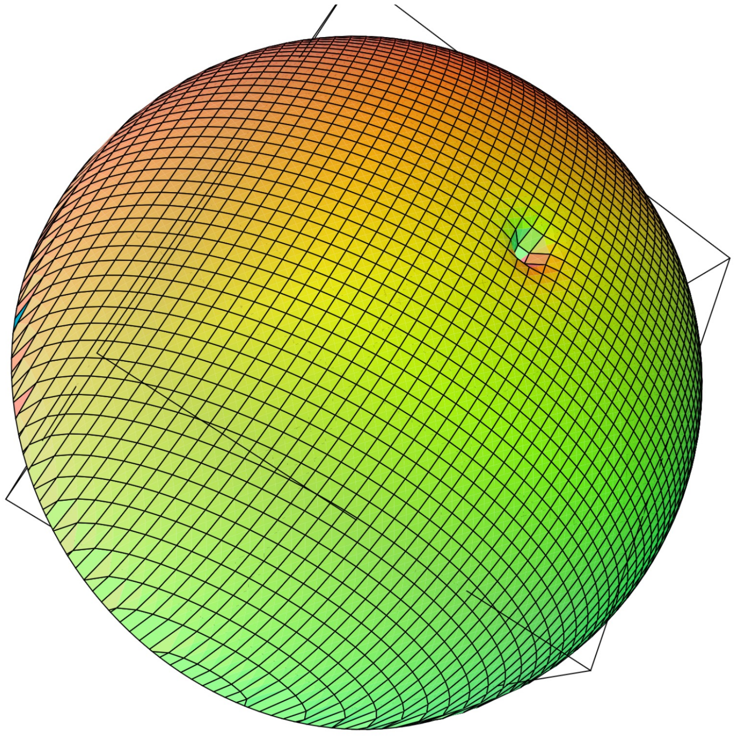
$$R^2(1 - \frac{r_g}{r})$$

$$\eta_1 = r\sin\theta\cos\phi, \quad \eta_2 = r\sin\theta\sin\phi, \quad \eta_3 = r\cos\theta,$$

$$\eta_4 = R\sqrt{1 - \frac{r_g}{r} - \frac{r^2}{R^2}}\sinh(\frac{ct}{R}), \; \eta_5 = R\sqrt{1 - \frac{r_g}{r} - \frac{r^2}{R^2}}\cosh(\frac{ct}{R})$$

Gravity Effect Simulation in 3D





$$x^2 + y^2 + z^2 = R^2(1 - \frac{0.01}{\sqrt{x^2 + y^2}})$$

Stereographic and 5-Dimensional

$$\eta_i = \frac{\xi_i}{1+\xi^2/4R^2}, \qquad \eta_5 = R\,\frac{1-\xi^2/4R^2}{1+\xi^2/4R^2}$$

5D Stereographic de Sitter Universe
Proves:

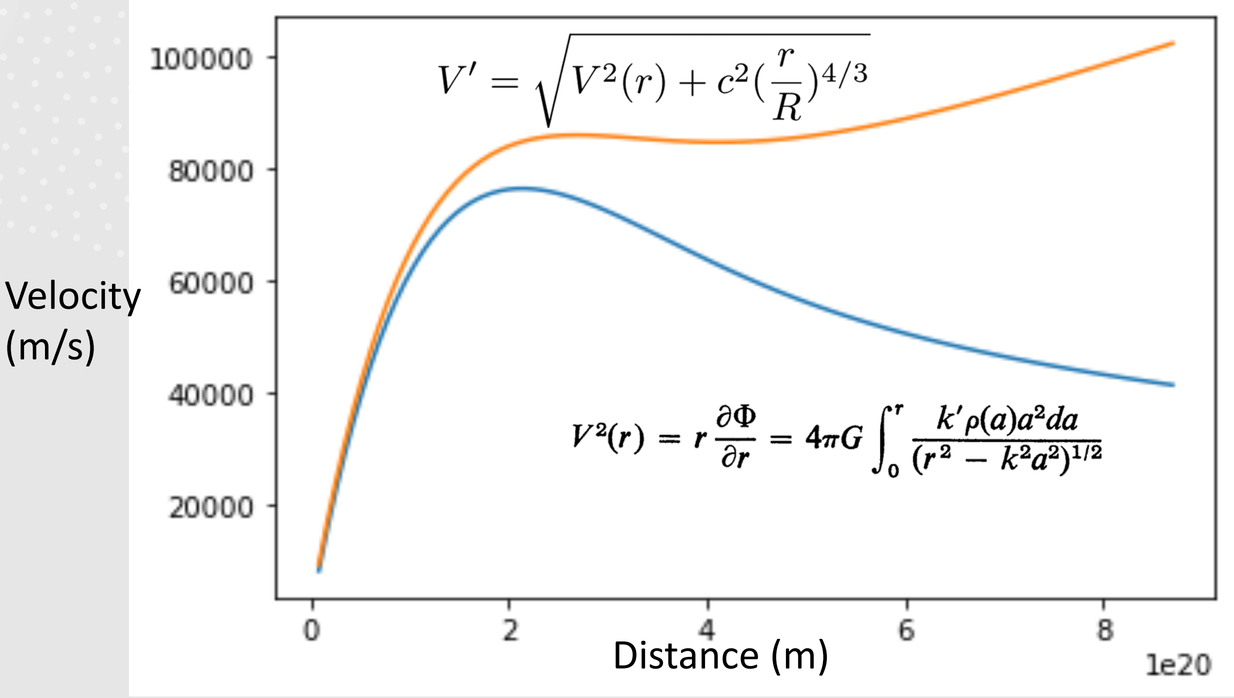
$$\eta_1^2 + \eta_2^2 + \eta_3^2 - \eta_4^2 + \eta_5^2 = R^2$$

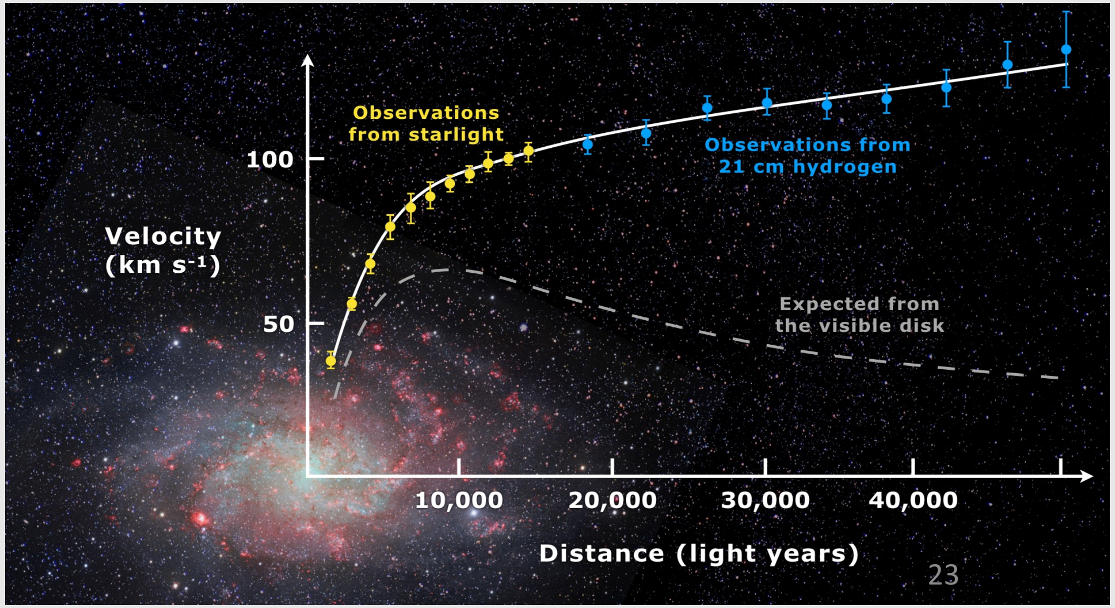
Phenomenology

- Galaxy rotation theory model:

$$V^2(r) = r\frac{\partial \Phi}{\partial r} = 4\pi G \int_0^r \frac{k'\rho(a)a^2da}{(r^2 - k^2a^2)^{1/2}}$$

Nordsieck, K. H. (1973)





Next Steps

- Derive a correction term to the Kottler metric and stereographic 5D spacetime.
- Refine phenomenology
- Calculate discrepancy between phenomenology and Kottler correction term
- Apply this hypothesis to other theories.

Images and Animations

https://en.wikipedia.org/wiki/Dark_matter

https://en.wikipedia.org/wiki/Galaxy_rotation_curve#cite_note-Rubin1980-15

https://simple.wikipedia.org/wiki/Dark_energy

https://c3d.libretexts.org/CalcPlot3D/index.html

References

Di Valentino, E., Melchiorri, A., Silk, J., “Planck evidence for a closed universe and a possible crisis for cosmology.” Nature Astronomy 4, 196 (2020).

Ferreira, P. G., “Cosmological tests of gravity.” Annual Review of Astronomy and Astrophysics 57, 335 (2019).

Liu, J., Chen, X., Ji, X., “Current status of direct dark matter detection experiments,” Nature Physics 13, 212 (2017).

Lord, E. A., “Tensors, Relativity and Cosmology” (McGraw-Hill 1976).

Milgrom, M. “A modification of the Newtonian dynamics as a possible alternative to the hidden mass hypothesis.” Astrophysical Journal 270, 365 (1983).

Rich, J., “Fundamentals of Cosmology” (Springer, 2001).

Brandt, J. C. (n.d.). *On the Distribution Of mass in Galaxies. I. The Large-Scale Structure of Ordinary Spirals with Applications to M 31.* 1960ApJ...131..293B page 294. http://articles.adsabs.harvard.edu//full/1960ApJ...131..293B/0000294.000.html.

Ryden, B., Peterson, B. M., “Foundations of Astrophysics” (Cambridge University Press, 2020).

Wesson, P. S., “Five-Dimensional Physics: Classical and Quantum Consequences of Kaluza-Klein Cosmology” (World Scientific, 2006).

Nordsieck, K. H. (1973). The angular momentum of spiral galaxies. methods of rotation-curve analysis. The Astrophysical Journal, 184, 719. https://doi.org/10.1086/152364

Bosma, A. (n.d.). The distribution and kinematics of neutral hydrogen in spiral galaxies of various MORPHOLOGICAL types - A. Bosma. http://ned.ipac.caltech.edu/level5/March05/Bosma/frames.html.

Gorbatenko, M. V., Sedov, S. Y. The Mannheim-Kazanas solution, the conformal geometrodynamics and the dark matter. arXiv.org. https://arxiv.org/abs/1711.06189. (2017)

Takeaways

- Theories of gravity might be modified.
- Our universe might be a 4D surface on a 5D sphere projected on a 4D plane.
- Dark energy and dark matter might be explained by this formalism along with stereographic projection.