University of New Haven

Dark Energy and Dark Matter as Five-Dimensional Stereographic Projection

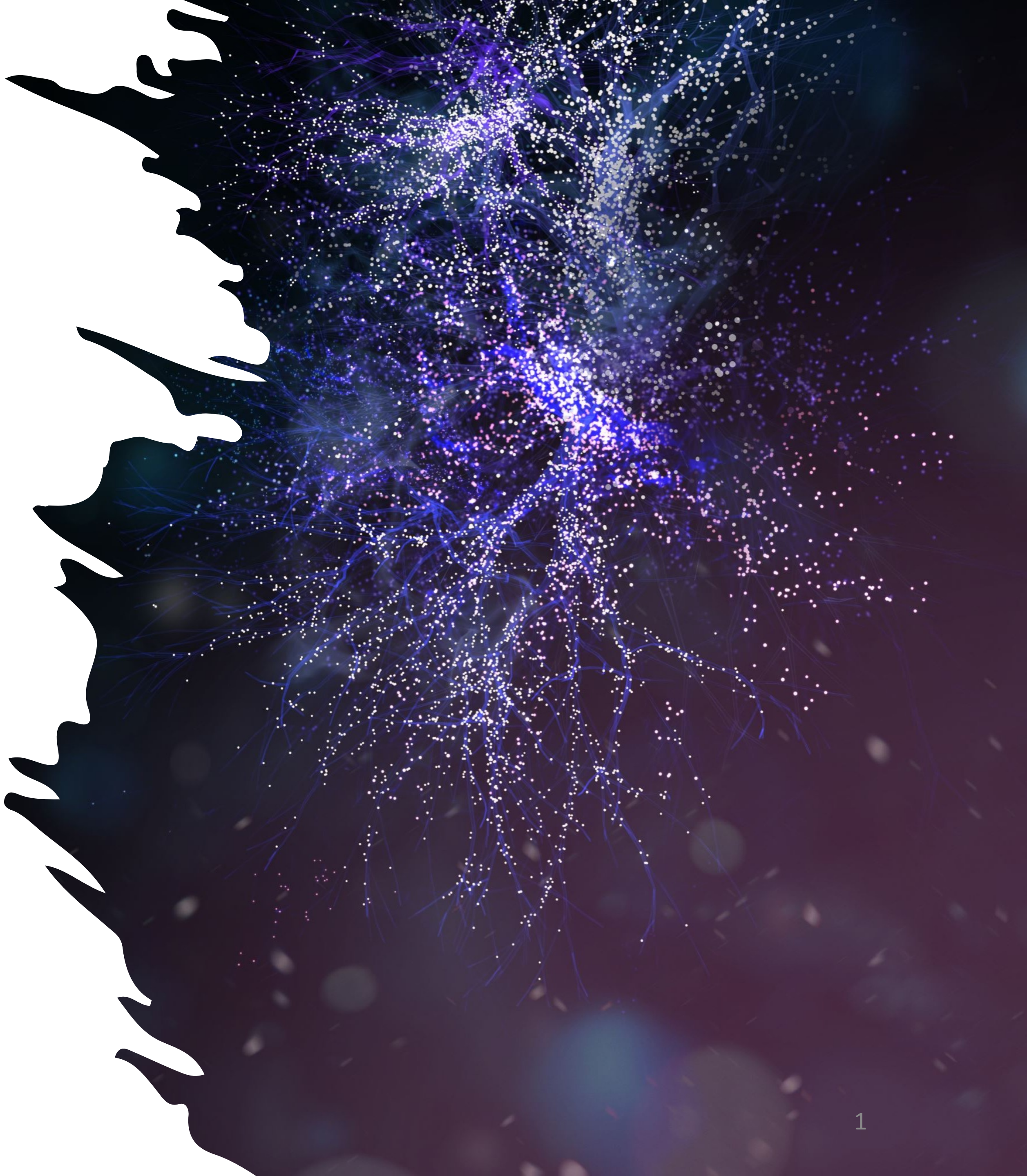
Hang Su

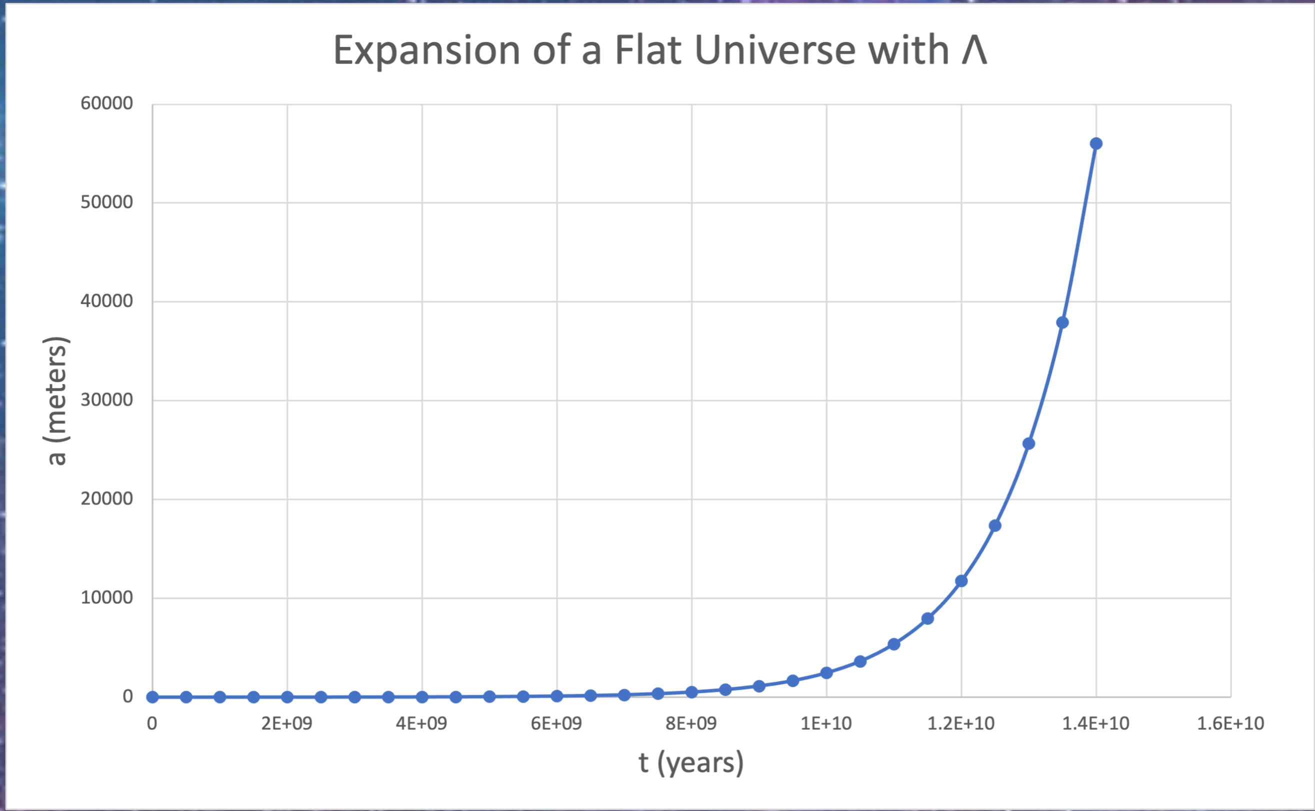
Nikodem Poplawski Ph.D.

Kevin Green Ph.D.

GCURS gcurs.rice.edu

RICE UNIVERSITY





$$\frac{da}{dt} = c\sqrt{\frac{\Lambda}{3}}a, \quad a = a(0)\exp\left(\sqrt{\frac{\Lambda}{3}}ct\right) = a(0)e^{Ht}$$

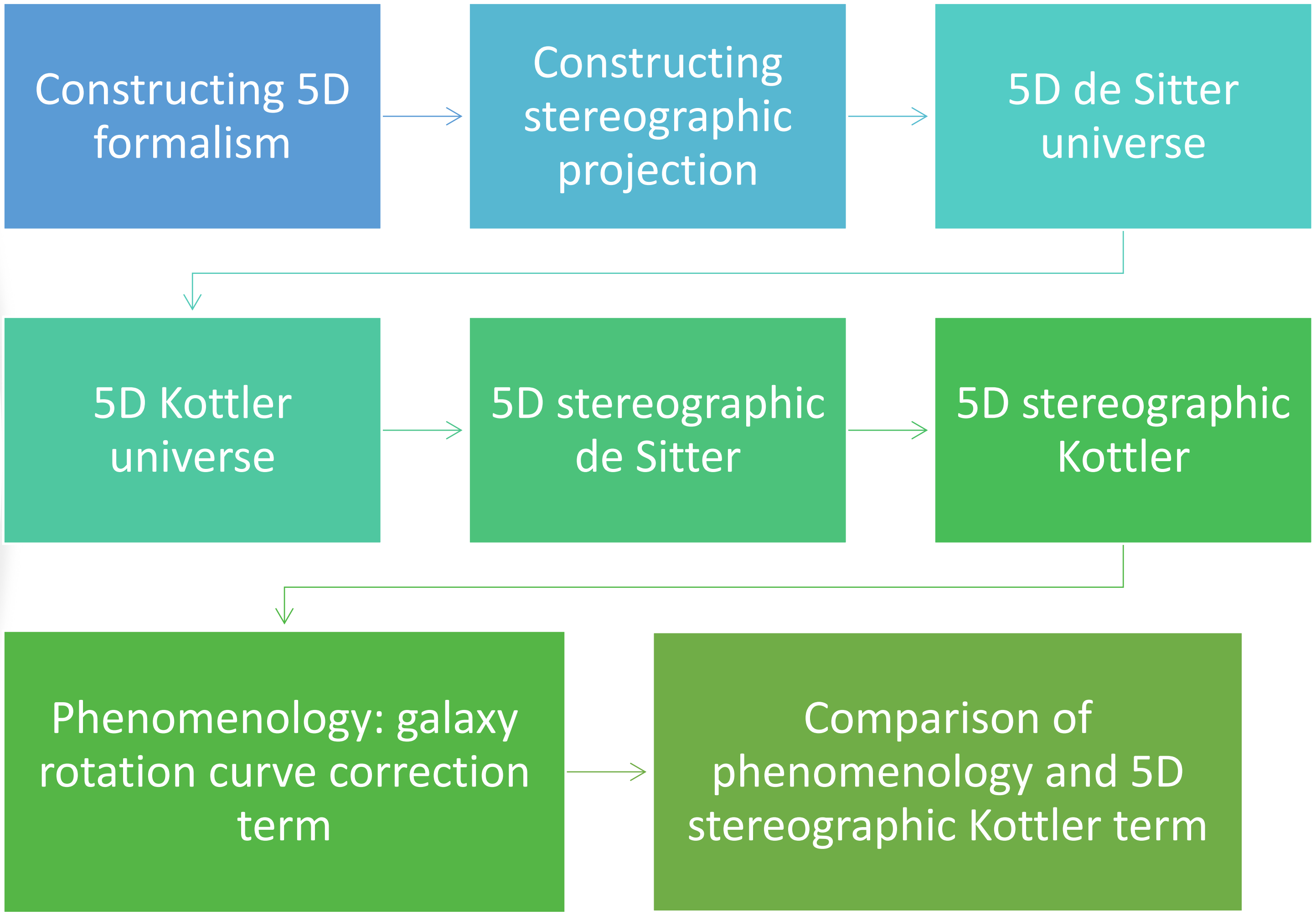
Empty universe with dark energy expands exponentially.

We hypothesize dark energy comes from the shape of the universe.

Hypothesis

Dark matter and dark energy are of the same nature, and they are the product of the universe being a 4D hypersurface on a 5D hypersphere projected onto a 4D hyperplane.

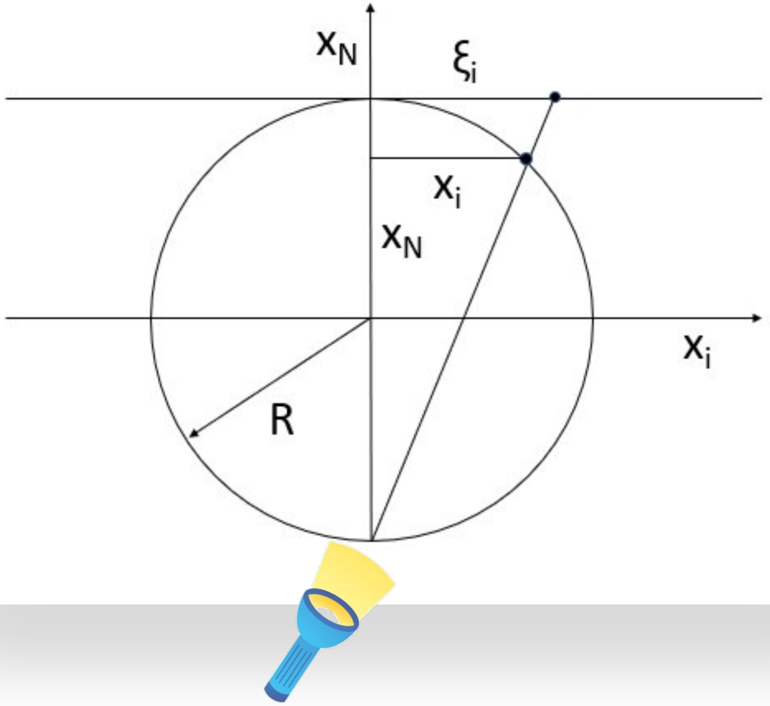
Methods


Constructing 5D formalism
Constructing stereographic projection
5D de Sitter universe
5D Kottler universe
5D stereographic de Sitter
5D stereographic Kottler
Phenomenology: galaxy rotation curve correction term
Comparison of phenomenology and 5D stereographic Kottler term

Preliminary Results

Stereographic Projection

$$\sum_i x_i x_i + x_N x_N = R^2$$



Stereographic Coordinates

- A point with coordinates $x_{i,}\, x_N$ $(i = 1, 2, N = 3)$ lies on a sphere with radius R.

$$x_i = \frac{\xi_i}{1 + \xi^2/4R^2}, \quad x_N = R\,\frac{1 - \xi^2/4R^2}{1 + \xi^2/4R^2} \qquad \xi^2 = \sum_i \xi_i \xi_i$$

The force that is believed to accelerate the expansion of the universe.

Dark Energy (Cosmological Constant Λ)

Empty universe looks like surface of 5D sphere with radius R in 5D pseudo-Euclidean, flat space:

$$\eta_1^2 + \eta_2^2 + \eta_3^2 - \eta_4^2 + \eta_5^2 = R^2$$

$$\eta_1 = r\sin\theta\cos\phi, \quad \eta_2 = r\sin\theta\sin\phi, \quad \eta_3 = r\cos\theta,$$

$$\eta_5 \pm \eta_4 = Re^{\pm ct/R}\left(1 - \frac{r^2}{R^2}\right)^{1/2},$$

This relationship gives:

$$\eta_1^2 + \eta_2^2 + \eta_3^2 = r^2, \quad \eta_5^2 - \eta_4^2 = R^2 - r^2$$

de Sitter metric:

$$ds^2 = c^2 dt^2 - a^2(0)e^{2Ht}(dr^2 + r^2 d\theta^2 + r^2 \sin^2\theta\, d\phi^2)$$

Coordinate transformation:

$$e^{2Ht} \to \left(1 - \frac{1}{3}\Lambda r^2\right)e^{2Ht} \qquad a(0)r \to re^{-Ht}$$

We get:

$$ds^2 = \left(1 - \frac{1}{3}\Lambda r^2\right)c^2 dt^2 - \left(1 - \frac{1}{3}\Lambda r^2\right)^{-1} dr^2 - r^2 d\theta^2 - r^2 \sin^2\theta \, d\phi^2$$

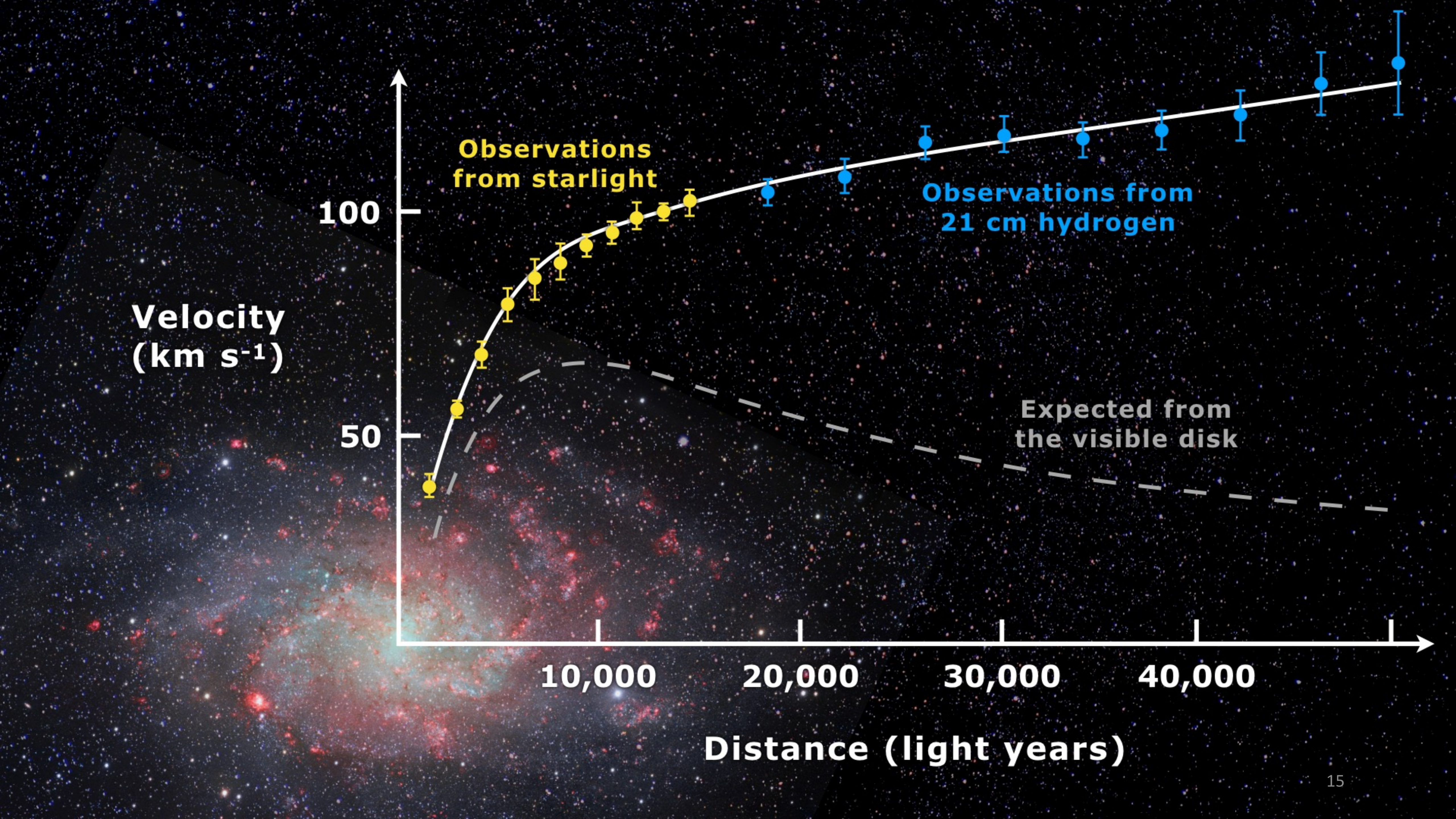
Compared to:

$$d\eta_1^2 + d\eta_2^2 + d\eta_3^2 - d\eta_4^2 + d\eta_5^2 = -\left(1 - \frac{r^2}{R^2}\right)c^2 dt^2 + \left(1 - \frac{r^2}{R^2}\right)^{-1} dr^2 + r^2 d\theta^2 + r^2 \sin^2\theta \, d\phi^2 = -ds^2$$

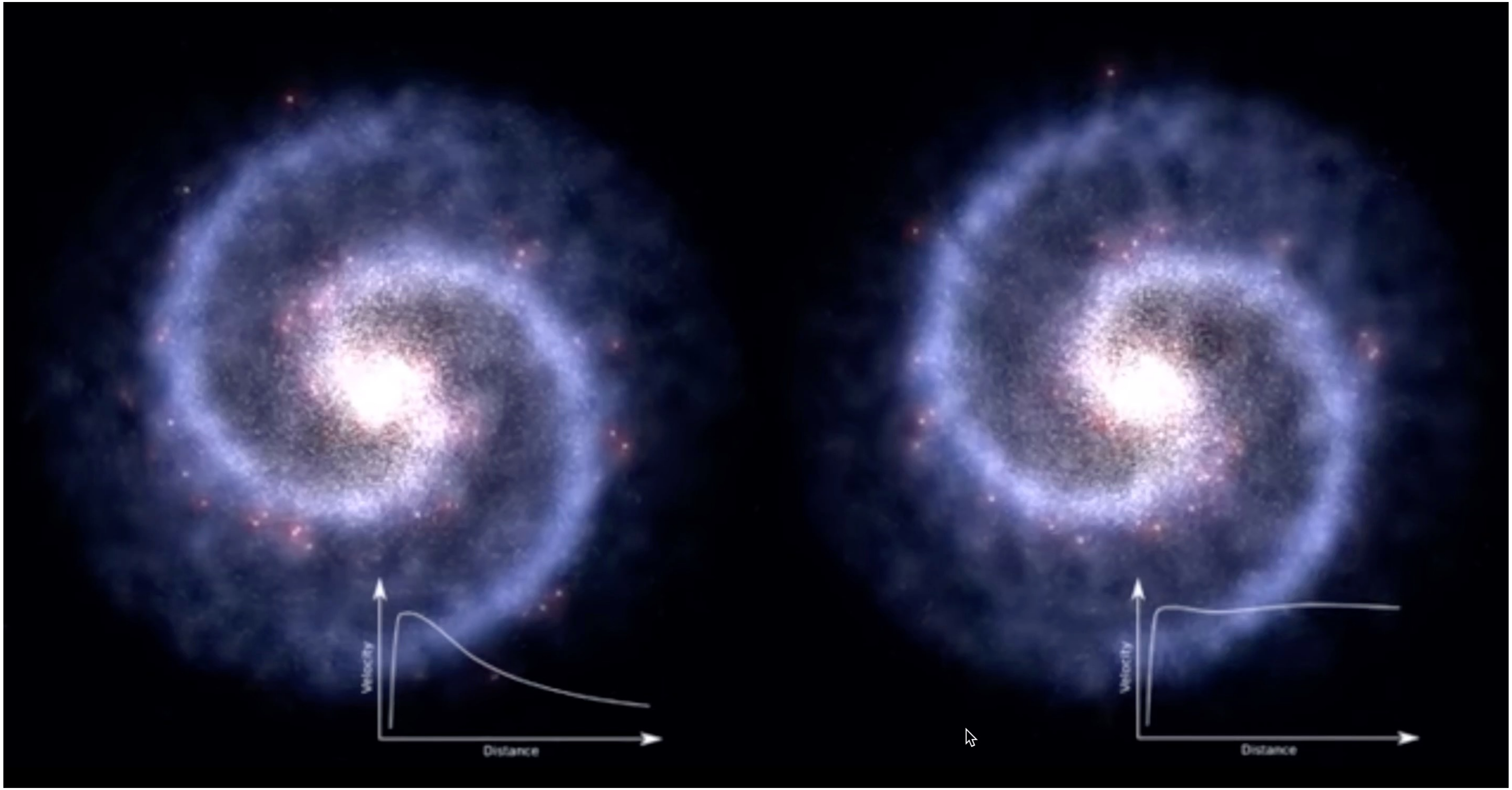
$$R = \left(\frac{3}{\Lambda}\right)^{1/2}$$

Dark energy is a nature of the 5D sphere.

Dark Matter



Observations from starlight
Observations from 21 cm hydrogen
100
50
Velocity (km s^{-1})
Expected from the visible disk
10,000
20,000
30,000
40,000
Distance (light years)


Velocity
Distance
Velocity
Distance

$$ds^2 = \left(1 - \frac{r_g}{r} - \frac{1}{3}\Lambda r^2\right)c^2 dt^2 - \left(1 - \frac{r_g}{r} - \frac{1}{3}\Lambda r^2\right)^{-1} dr^2 - r^2 d\theta^2 - r^2 \sin^2\theta \, d\phi^2$$

Kottler Metric of Spacetime

- Kottler: Schwarzschild-de Sitter universe

Kottler in 5D

$$d\eta_1^2 + d\eta_2^2 + d\eta_3^2 - d\eta_4^2 + d\eta_5^2 = -ds^2 + dr^2\left(\frac{\frac{R^2 r_g^2}{4r^4} - \frac{2r_g}{r}}{1 - \frac{r_g}{r} - \frac{r^2}{R^2}}\right)$$

de Sitter in 5D

$$d\eta_1^2 + d\eta_2^2 + d\eta_3^2 - d\eta_4^2 + d\eta_5^2 = -ds^2$$

Prove: Kottler universe is a 4D surface of a 5D deformed sphere in not flat 5D space.

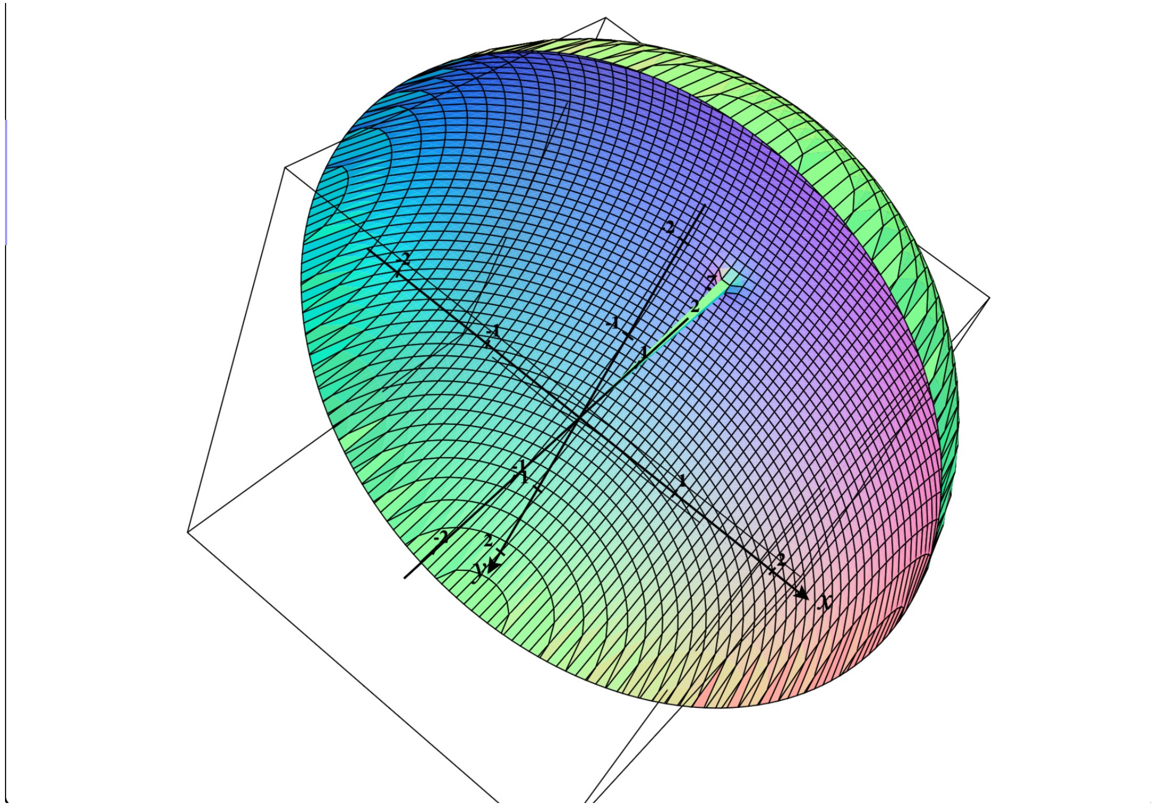
$$\eta_1^2 + \eta_2^2 + \eta_3^2 - \eta_4^2 + \eta_5^2 =$$

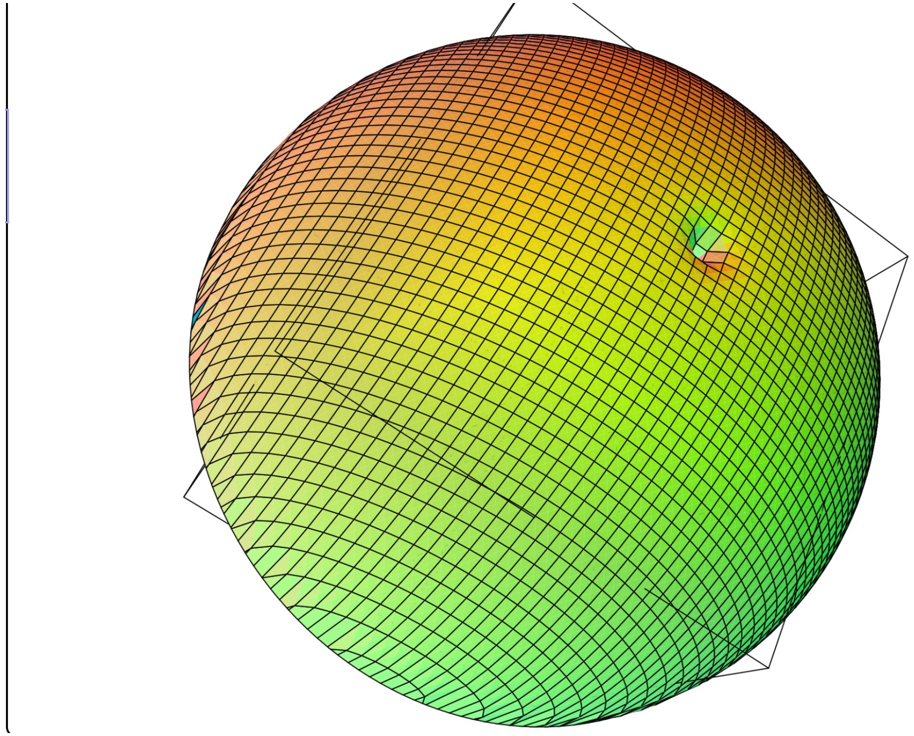
$$R^2(1 - \frac{r_g}{r})$$

$$\eta_1 = r \sin\theta \cos\phi, \quad \eta_2 = r \sin\theta \sin\phi, \quad \eta_3 = r\cos\theta,$$

$$\eta_4 = R\sqrt{1 - \frac{r_g}{r} - \frac{r^2}{R^2}} \sinh(\frac{ct}{R}), \ \eta_5 = R\sqrt{1 - \frac{r_g}{r} - \frac{r^2}{R^2}} \cosh(\frac{ct}{R})$$

Gravity Effect Simulation in 3D





$$x^2 + y^2 + z^2 = R^2\left(1 - \frac{0.01}{\sqrt{x^2 + y^2}}\right)$$

Stereographic and 5-Dimensional

$$\eta_i = \frac{\xi_i}{1+\xi^2/4R^2}, \quad \eta_5 = R\,\frac{1-\xi^2/4R^2}{1+\xi^2/4R^2}$$

5D Stereographic de Sitter Universe Proves:

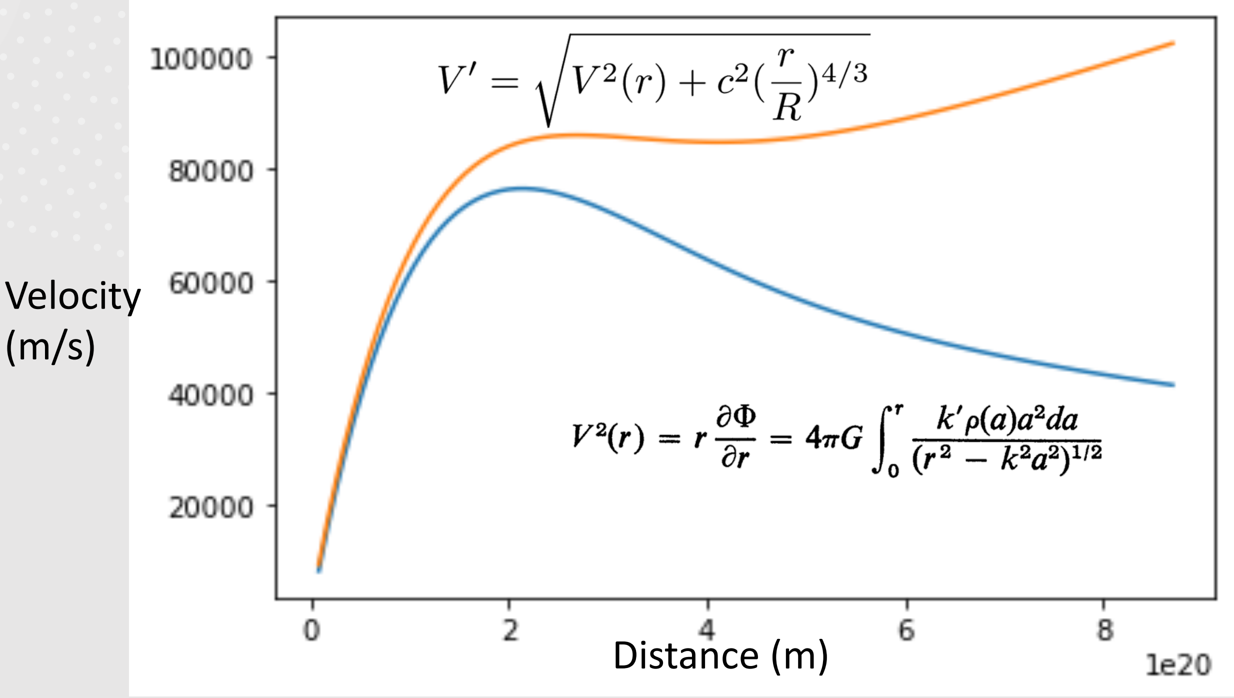
$$\eta_1^2 + \eta_2^2 + \eta_3^2 - \eta_4^2 + \eta_5^2 = R^2$$

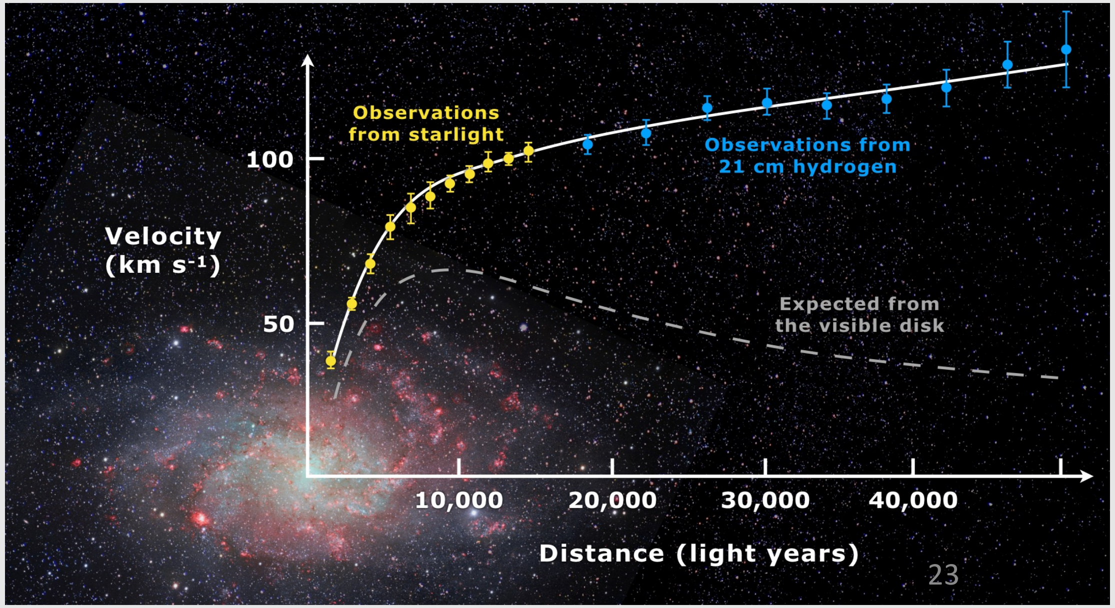
Phenomenology

- Galaxy rotation model:

$$V^2(r) = r\frac{\partial \Phi}{\partial r} = 4\pi G \int_0^r \frac{k'\rho(a)a^2 da}{(r^2 - k^2a^2)^{1/2}}$$

Nordsieck, K. H. (1973)





Next Steps

- Derive a correction term to the Kottler metric and stereographic 5D spacetime.
- Refine phenomenology
- Calculate discrepancy between phenomenology and Kottler correction term
- Apply this hypothesis to other theories.

Images and Animations

https://en.wikipedia.org/wiki/Dark_matter

https://en.wikipedia.org/wiki/Galaxy_rotation_curve#cite_note-Rubin1980-15

https://simple.wikipedia.org/wiki/Dark_energy

https://c3d.libretexts.org/CalcPlot3D/index.html

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Takeaways

- Theories of gravity might be modified.
- Our universe might be a 4D surface on a 5D sphere projected on a 4D plane.
- Dark energy and dark matter might be explained by this formalism along with stereographic projection.